Wittgenstein for Beginners

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Ludwig Wittgenstein (1889-1951) was an Austrian-British philosopher of partial Jewish descent and considered by many the greatest philosopher of the 20th century. He was influenced by Emanuel Kant and was a student of Gottlob Frege and Bertrand Russell and influenced many others.

Wittgenstein worked primarily in logic, the philosophy of mathematics, the philosophy of mind, and the philosophy of language. His obscured and incomprehensible ideas to most mortals have been given a wide range of interpretations. The author of this article also inserted a few of his personal views between the lines.

In this article would be presented a few interwoven topics: What is required from a proposition to claim something about the world. Is Mathematics Discovered or Invented? Are the limits of language the limits of our world? An interesting anecdote, – for the end.

True or False

Wittgenstein introduced his basic concepts of logic theory in his first and only book published during his lifetime – *Tractatus Logico-Philosophicus* (*Latin for: Logical Philosophical Treatise*), published in 1921.

The main purpose of this book is to identify the relationship between language and reality and especially to find out what it takes for a proposition to claim something about the world. According to Wittgenstein, a real proposition can be only empirical (derived from reality or experience) that is true if it agrees with reality or false if not. It follows that the other types of propositions (non-empirical) are unreal and their inherent truth or falsehood are different from the true and false of empirical propositions.

For example, the proposition: "It is raining now" is empirical because we can open the window and find out easily if it is indeed raining, or we can even use measuring instruments for this end. According to Wittgenstein, if it does rain then the proposition is true and if it does not rain then the proposition is false.

On the other hand, the proposition "all singles are unmarried" is based on arbitrary definitions of the terms "single" and "married", which is always correct by definition and self-evident and in fact it is a repetition of the same in other words that says nothing about the world. According to Wittgenstein, since the proposition "all singles are unmarried" does not require empirical confirmation (any empirical test, in this case, is meaningless, see below) it cannot be defined as true but only as making sense (sensical) since being a product of the human mind it has an understood meaning but no more than that. According to this approach the opposite proposition "all singles are married" is always incorrect by definition and is not making sense (nonsensical) but not false. This kind of proposition is based on the definition of contradictory terms, is always incorrect and it also says nothing about the world.

Sensical propositions and nonsensical propositions do not depend on any empirical test for validation and are, therefore, not a necessary consequence of reality. In fact, such propositions are an invention of the human mind and their existence depends solely on it, and it is clear that "true" and "false" do not apply in these cases but only sense or nonsense.

True and false are values that rely on reality while sense and nonsense are derived exclusively from the mind.

There are several types of non-empirical propositions:

Tautology – a proposition that is self evident and always correct as "all singles are unmarried".

Contradiction – a proposition that is always incorrect by definition as "all singles are married".

Subjective proposition – a proposition that cannot be proved or refuted empirically or by the mind. For example: religious beliefs (there is only one God!); Ethics (stealing is wrong); Aesthetics (Diana is a beautiful girl); etc.

Wittgenstein maintains that the expression "2 + 2 = 4" does not describe an empirical mathematical reality that can be defined as "true" but only as sensical that is always correct by definition (tautology) when using mathematical terms (numbers and arithmetic operations) agreed upon, and that is because this operation is defined arbitrarily as correct like any other linguistic grammatical law. It follows that "2 + 2 = 5" is not "false" but "nonsense" just like the contradiction "all singles are married".

We could define arbitrarily that "1 + 1 = 3" and if there is general agreement upon this, we got a sensical tautology that makes sense and then the one familiar till now "1 + 1 = 2" would be a nonsensical contradiction.

Let us take a look at a familiar theorem of plane geometry: If three pairs of sides of two triangles are equal in length, then the triangles are congruent. This theorem is a tautology because it is always correct in all circumstances and cannot be put to the empirical test because it is an outcome or invention of the human mind (see below).

An empirical test means an observation that tests empirically a proposition that may be "true" under certain conditions and "false" under other conditions. But if the observation gives always the same result then an empirical test is meaningless. This is exactly the case with congruent triangles that always are congruent as long as they obey congruence theorems. There is no situation by which two triangles will be discovered in nature that obey any congruence theorem and will not be congruent. Therefore the congruence theorems (and other geometry theorems) could not be subjected to empirical tests since they are the result of the human mind and they can be sensical but not "true" or "false". But, nevertheless, geometry and mathematical theorems say nothing about the world, like other tautologies and contradictions, they are useful in quantification and they allow the formalization, by using formulas, of natural phenomena.

Let's test empirically the physical phenomenon that water boils at 100 ° C. Many are going to say that since water always boils at 100 ° C, then this fact is a tautology and therefore an empirical test is meaningless, but this is not the case. Because the boiling point depends on the atmospheric pressure (barometric pressure) at which the process takes place – the lower the atmospheric pressure, the lower the boiling point. For example, the boiling temperature of water at sea level is 100°C, rising at the Dead Sea to 101°C and falling back to 71°C at the Everest top. When the boiling point of a particular liquid is indicated, it refers to the boiling point at sea level where the atmospheric pressure is one atmosphere. It is clear that if the conditions of Earth's atmospheric pressure will change one day, for some reason, then the boiling point of water will also change accordingly at sea level. Triangles, on the other hand, will always be congruent anywhere in the universe and under any conditions and circumstances.

Experimenting with the boiling point of water, mentioned above, is a scientific experiment and as a matter of fact, any scientific experiment is an empirical test.

According to Wittgenstein, a logical proposition is a proposition that can be valued as true or false, like the empirical propositions. Even the sentences describing sensical tautologies and nonsensical contradictions are still logical because although true or false values cannot be attributed to them since they are not dependent on any empirical reality, these propositions are a product of reason and still have some practical use (to find out whether someone is single or not; whether two triangles are congruent). Logical propositions could be talked about (the tautologies and contradictions, hardly) but beyond them, there are the subjective, illogical propositions, to which true or false values cannot be attributed and their validity or invalidity cannot be put to any empirical test or reasoned by the mind like tautologies and contradictions, because subjective propositions do not represent a logical picture in our world (I cannot empirically check whether God exists or does not exist nor prove or contradict it through reason – Kant showed this in his book **Critique of Pure Reason**). Nor can I put to an empirical test the proposition: "Diana is a beautiful girl" nor can I prove it or contradict it through reason (all in the eye of the beholder).

According to Wittgenstein humans can think only logically and only the logical exists in our world. But Wittgenstein's ideas are even more far-reaching because in his view we can speak and think only about logical propositions. But the most substantive and important issues of human life are concerning morality (ethics), religion, aesthetics, metaphysics and the essence of life which are subjective values and Ideas – the mystical – and therefore lack a logical basis and as such are outside the realm of human cognition and beyond language or what can be spoken about. Hence the famous conclusion of his *Tractatus Logico-Philosophicus*: "Whereof one cannot speak, thereof one must be silent." (See below.)

Wittgenstein was a religious person (Christian) who believed in the existence of God, what seemed to contradict his own philosophy, which views God as an illogical entity that could not exist in our world, but he was aware of the limitations of his faith and labeled it as mysticism.

According to Kant, the human mind renders a subjective picture of the world but it is the only tool we have for knowing and realizing our world. Our reality is what this tool is capable to recognize. The human brain works the way it works, which we call, for the sake of convenience, logic and therefore the limits of logic are the limits of our consciousness and our reality since that is what the human mind renders. Wittgenstein acknowledges this fact and argues that everything that is not logical cannot exist in our world and in reality and therefore it is a waste of time to talk about them. All those who think that there are things beyond logic, like faith, aesthetics and morality actually claim that the human mind is capable to comprehend what it is unable to comprehend. Rational thinking means awareness of the limits of the human mind.

Wittgenstein's goal in setting an empirical touchstone or an objective verification principle in the form of reality itself to determine the validity of propositions, was to separate the valid from the invalid, by removing ambiguity and confusion, and placing logic on solid foundations, otherwise logic wouldn't have any objective meaning within the subjectivity of the human mind.

Wittgenstein called himself "the last of the philosophers" because he thought he had solved in his *Tractatus* all the problems of philosophy and even retired from studying philosophy as a result.

Is Mathematics Discovered or Invented?

Wittgenstein argues that mathematics is an invention rather than a discovery and as a matter of fact the mathematician is an inventor. Nothing mathematically exists unless humans invented it.

According to Wittgenstein, mathematical propositions are not empirical and the "mathematical truth" is not empirical but the fruit of the human mind. Man developed mathematics through calculations and mathematical proofs. Although one can learn from a proof that a mathematical theorem can be deduced from axioms and by employing other mathematical theorems in a certain way, it is clear that this proof had not existed before it was developed by a mathematician and therefore it is obvious that it is not empirical but a product of the human mind – an invention.

Is it possible that the day comes when a mathematician will announce the world enthusiastically that "2 + 2 = 11" and also prove it? If this happens we are going to admit that mathematics is an invention rather than a discovery since before that we only knew that "2 + 2 = 4". But this has already happened before, and it will happen again (see below).

Most people if asked why they believe that "2 + 2 = 4" is an absolute truth are going to say that every time we take two apples and put them together with more two apples in a basket we get always four apples. Only few will admit that this is an arbitrary game with agreed-upon signs concocted by the human mind.

That is, in nature:

(apple, apple, apple, apple) is only (apple, apple, apple, apple), and nothing else.

But if:

(apple, apple, apple, apple) = 2 apples + 2 apples, or if:

(apple, apple, apple, apple) = (apple + apple + apple + apple), or if:

(apple, apple, apple, apple) = 4 apples, or if:

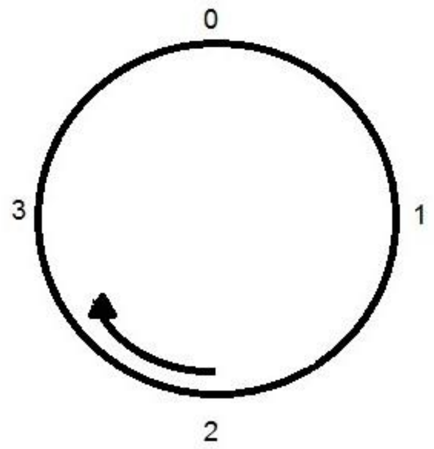
(apple, apple, apple, apple) = 1 X 4 apples, then:

This is already an invention of the natural numbers and arithmetic operations.

Man, in an arbitrary and agreed-upon way, invented the common addition and subtraction operations that are expressed by a left or by a right movement on a horizontal number line. If we want to know how much is "2 + 2" then we have to move to the right 2 points from the point representing "2" on the number line and we get 4. In the same way, regarding the calculation of "2-1" we move from 2 one point to the left and get the correct result 1.

Out of arbitrary convenience, we may decide that in order to perform the addition and subtraction operations we do not move right or left on a straight line but instead move on a circle circumference divided by points representing numbers – what is called modular arithmetic.

If the circumference of a circle is divided by four points (modulo 4), it is easy to realize that according to this method "2 + 2 = 0" because by moving two points from 2 clockwise on the circle circumference we reach "0". The interesting thing about this method is that if we subtract by counterclockwise movement then also "2 - 2 = 0".



modulo 4

In modulo 12, like a regular clock, although "2 + 2 = 4" but "11 + 2 = 1".

Modular arithmetic has many uses in computer science, banking, music and more.

Numeral systems are also a matter of arbitrary convenience and there is no objective qualitative advantage of the familiar decimal numeral system over the others. In the decimal system "2 + 2 = 4" but in base 3 number system it turns out that "2 + 2 = 11" (the dramatic announcement to the world, remember?) while in the binary system "2 + 2" does not exist.

In Boolean algebra indeed "1 x 0 = 0" but "1 + 1 = 1".

The examples given above are about addition of numbers like in basic regular addition (besides Boolean algebra), where we add magnitudes of numbers, what we call scalar addition. There is also in existence a vector addition operation of physical or mathematical quantities that takes into account not only magnitude but also the direction or angle between the quantities, using the triangle or the parallelogram methods.

The same applies in the field of geometry. We have learned at school that the sum of the angles of a triangle is always equal to 180 degrees (Euclidean geometry). But according to the hyperbolic geometry of Lobachevsky and Bolyai the parallel postulate is different than in Euclidean geometry and it follows that the Sum of angles of a triangle is always less than 180 degrees. Whereas according to Riemann's spherical geometry there are no parallel lines at all and the sum of the degrees of a triangle is always greater than 180. There are also various other non-Euclidean geometries that are equally valid and used in relativity, astronomy, navigation and more.

All the methods of performing an addition operation presented above (and there are more) are equally valid even though they give different results for the same operation.

2 + 2 = 4	– decimal numeral system
2 + 2 = 0	– modulo 4
2 + 2 = 11	– base 3
2 + 2 = 10	– base 4
2 + 2 does not exist	– Boolean algebra
2 + 2 depends on direction	– vector addition

The same multitude applies in geometry – which implies the arbitrariness employed by the human brain and its inventive virtuosity rather than an empirical reality derived from experience, what results in various mathematical and geometrical systems.

It is possible to argue that the multitude of different arithmetical and geometrical theories and systems with same validity are not due to invention but they all describe the physical world from different existing aspects and different points of view because the complexity of the world.

It should be borne in mind that certain fields of mathematics are abstract, purely theoretical and achieved by reason, like certain branches of number theory and geometry, which have no practical use and any theoretical reference to the physical world and probably will never have. All this reinforces the invention option over the discovery in mathematics.

The question whether the development of mathematics, as expressed in suggesting a new conjecture or in finding a new proof, is a discovery or an invention, occupied mathematicians in the late 19th and early 20th centuries, although its roots go as far as to Aristotle and Plato.

On the one hand, some argue that all mathematical objects (theorems, proofs, etc.), those we know and those we do not know, exist in some "virtual space", and all that is left is to discover them. According to this approach, the formulation and proof of a new theorem is a discovery. Accordingly, the development of mathematics is nothing but the development of human knowledge about mathematics. Even though a mathematical theory is not yet known, it exists in the first place, and imposes the path that must be followed in order to discover it.

Many, including Wittgenstein, do not accept this approach, because it is clear that the "discovery" of the proof to Fermat's Last Theorem is different from the discovery of an obscure island in the ocean or the discovery of a plant that was not previously known. The proof of Fermat's Last Theorem involves a great deal of creativity, and to claim that the proof has always existed and it only had waited to be excavated is not far from claiming that a new poem is not the creativity work of a poet but a discovery of the poem in the wide sea of all phrases and possible existing rhymes, or that a sculpture lies in the marble block in the first place and the sculptor only excavates it by removing the unnecessary debris from the marble block using the hammer and chisel. According to this, one can claim that every invention is in fact a discovery – Alexander Bell did not invent the telephone but discovered it because all the physical materials and scientific principles that underpinned his invention existed long ago and he only excavated them and connected them in a certain way he discovered that had already existed forever and waited to be exposed. Suppose that a mathematical theory has indeed not yet been discovered – does that also mean that it really exists? If a tree falls in a forest and no one is around to hear it, does it make a sound?

According to this approach, that Wittgenstein represents, mathematics is a creation of the human mind, and in fact mathematics is a man-made invention.

But how was mathematics invented? It seems that about 8000 years ago, when an ancient Middle Eastern shepherd wanted to make sure that the entire herd returned from grazing, he began counting his sheep with the help of small pebbles. In this action, in fact, the anonymous shepherd invented the natural numbers and also the addition and subtraction operations that did not exist before his time on earth and which he concocted in his mind – the rest is history.

Herds of sheep grouped from individual heads existed before the invention of mathematics and sheep from the herd disappeared even before that and occasionally the shepherd could spot this visually and intuitively. Although the tool for accurate quantification of the phenomenon (mathematics) had not yet existed, the mathematical phenomenon in nature, the addition and subtraction operations, as learned in elementary school, had existed in principle anchored in everyday life. "2 + 2 = 4" did not exist formally but was embodied and realized in practice in the act of nature and reality. If this is so, it could be argued that the clock is also actually a discovery and not an invention since it quantifies the flow of time that existed before man appeared on earth.

But how come that mathematics, which is the fruit of human thought, independent of experience and observation, reflects so beautifully the physical reality in our world?

According to Kant, mathematical propositions are informative but do not provide information about the world itself, but they do provide information about the world as it is perceived in our experience, that is, the world through the glasses of Pure Reason. Mathematics is not the rules that govern the world but the laws of logic and reason through which the human mind perceives the world and organizes the information gathered by the senses from the outer world into a consistent and subjective experience of reality.

Simply put, according to Kant, the same brain that invented mathematics is also the same brain that is responsible for a person's subjective perception of the world and since it is the very same machine and the same logic in both cases, the results are also compatible.

For example: The invention of the telephone, which is the fruit of the human mind, also functions well in reality.

Language and Reality

Wittgenstein's great innovation, in the field of philosophy of language, is the assertion that there is no thinking without language. Descartes, on the other hand, argued that words are not necessary in order to think and hence his famous phrase: "I think, therefore I am." Wittgenstein would have phrased this sentence differently: "I have language, I think, therefore I am."

In Wittgenstein's opinion language shapes consciousness and words reflect reality and are a logical picture of it. A similar view was expressed by George Orwell in his book "1984" where citizens live in a totalitarian state where everyone is under the watchful eye of "Big Brother". History is rewritten according to the needs of the leadership, and the language spoken is adapted so that the thinking ability of the citizens is limited, by reducing vocabulary by eliminating words like "freedom" – if the word "freedom" does not exist in the vocabulary then there is also no freedom, in reality, that humans can aspire to. Hence Wittgenstein's revolutionary conclusion that the limits of language are the limits of thought, consciousness, and reality.

It could be deduced from Wittgenstein's views that mathematics is actually another language that allows us to think about reality through numbers and agreed-upon signs that are in fact the words that make up this language.

In his second important book, *Philosophical Investigations*, Wittgenstein asserts that it is not necessary to define concepts or words precisely but to understand them intuitively according to the practical use made of them in actual daily life.

If someone says that "Diana is a beautiful girl" (subjective, illogical proposition) one can agree or deny this statement or not express his opinion at all, but it would be counterproductive to try to find out what "beautiful" means as it will lead the discussion into a dead end meaningless and detrimental to the subject in question. There is no need to delve into the essence of "beautiful" and the concept must be understood and used in the simple intuitive way which is clear for everyone without adding unnecessary interpretations – another reason for "Whereof one cannot speak, thereof one must be silent".

According to Wittgenstein, many of these problems arise through an inflexible view of language that insists that if a word has a meaning there must be some kind of object corresponding to it. Thus, for example, we use the word "mind" without any difficulty until we ask ourselves "What is the mind?" We then imagine that this question has to be answered by identifying some "thing" that is the mind. If we remind ourselves that language has many uses and that words can be used quite meaningfully without corresponding to things and definitions, the problem disappears.

In fact, this was the method which Socrates employed, in the Platonic dialogues, in his discussions by asking passersby on the streets of Athens to clarify the meaning of concepts and thus confused and embarrassed them showing that they actually do not know what they are speaking about.

Anecdote

It turns out that Wittgenstein and Hitler attended the same high school in Linz, Austria. Since they were both born in 1889 six days apart, the piquant question arises, whether they also studied in the same class, and what was the nature of their relationship. As far as is known, Hitler repeated a year while Wittgenstein skipped one so that two grades separated between them, nor is there any evidence that there was any other relationship between them. Others present a picture showing Hitler and Wittgenstein together but the picture appears to have been taken before Wittgenstein arrived at the school. There were those who claimed that the wealthy genius rich Jew Wittgenstein fed Hitler anti-Semitism but this is apparently unfounded speculation.

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